Dynamic magnetic scalar hysteresis lump model, based on Preisach model quasi-static contribution extended with dynamic fractional derivation contribution

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Accurate and simple magnetic material law is necessary to correctly model complete electromagnetic systems. In this article, a new formulation based on the scalar quasi-static hysteresis Preisach model extended to dynamic behavior using fractional derivation dynamic contribution is proposed. The fractional contribution is solved using convolution which highly reduces the numerical issues. The order of the fractional derivation provide a new degree of freedom and allows to correctly obtain simulation results on a very large frequency bandwidth. By using such formulation, space discretization techniques (finite differences, finite elements) are avoided which are highly space and time consuming while keeping the global simulation results precise. The numerical implementation of the problem and some experimental validations are shown in the article.

***Index Terms*— magnetic hysteresis, fractional calculus, electromagnetic modeling.**

I. INTRODUCTION

The development of new electromagnetic designs, such as the improvement of already existing ones require precise simulation tools. Similar tools can also be used for the understanding and interpretation of non-destructive eddy current testing and Barkhausen noise measurements' electrical signatures. Recent scientific investigations based on such numerical tools mainly focus on coupling Space Discretization Techniques (SDT)(Finite Elements Method (FEM), Finite Differences Method (DFM)) extended with accurate scalar or vectorial, dynamic or static, and considering hysteresis material law. For this magnetic material law, it seems that the best results come from the extension of the quasi-static hysteresis model (Preisach model [1], Jiles-Atherton model [2]) to dynamic behavior as a result of the separation losses techniques as proposed by Bertotti [3]. The simultaneous resolution between SDT procedures and hysteresis models can be realized by iterative techniques. One of them is the so-called fixed point scheme [4]. Such improved SDT give very accurate results, unfortunately the high nonlinear behavior of hysteresis often leads to uncertain convergences which leads to numerical errors [5]. Furthermore SDT requires huge memory space and such simulations are always highly time consuming. In this article, an original alternative to scalar SDT is proposed. In this approach, time and memory space consuming SDT is replaced by the lump model while keeping very accurate simulation results on a large frequency bandwidth. Fractional time derivative operators makes it convenient to incorporate the lump model. For this new approach, two contributions are required:

- a quasi-static contribution using Preisach model.
- a dynamic contribution using fractional time derivation of the magnetic induction field.

Different definitions of the fractional derivation are available in the literature. In this work resolution by convolution has been chosen which simplifies the numerical scheme and reduce the computation times.

II. MODEL

A. *Quasi-static contribution*

Due to the domain wall movements, microscopic eddy currents generate through the cross section of a magnetic sample as soon as it is exposed to a varying magnetic field. Below a threshold frequency (in the decreasing direction) the cumulative periodic value of these microscopic eddy currents become frequency independent. This behavior is called as the quasi-static state. This quasi-static contribution is observable when plotting the spontaneous average magnetic induction field, B, versus the surface magnetic excitation field, H for very low frequency (f<<1Hz in typical soft magnetic material). Different approaches are available in the literature for the simulation of the quasi-static hysteresis behavior [1]. Among all, Preisach's model exhibits the interesting property of being easily invertible. It is indeed relatively easy to switch from H to B as input of the quasi-static hysteresis model. Preisach's model has been widely used to describe the hysteresis phenomenon in magnetic materials [6]. It assumes that the material magnetization is determined by the contribution of a set of elementary hysteresis loops having a distribution function over the Preisach's triangle. In order to model precisely the magnetic material behavior, it is necessary to accurately determine the distribution function from experimental data. There are mainly two ways to determine this distribution function. The first way assumes that the distribution function has a particular form (Lorentzian, Gaussian) and then determines the parameters of the chosen function in order to depict the average hysteretic behavior. The second way discretizes the distribution function in a finite set of values which are determined by suitable experimental data. In this study, the second option is chosen which is expected to provide higher accuracy.

Two techniques have been tested for the acquisition of the discretized distribution function: The centered cycle technique as described in [6] and the Biorci's method [7]. Both techniques provide relatively correct simulation results. A higher number of experimental data is required by the first technique but a correct behavior can be reached with a lower size of the discretized distribution implementation and this means a simpler and reduced memory management.

B. Dynamic contribution

Under weak frequency conditions, scalar quasi-static lump hysteresis models provide accurate results for the evolution of the average magnetic induction B versus magnetic excitation field H. Such particular external conditions mean homogeneous distributions of the induction through the cross section of the sample and consequently homogeneous distributions of the magnetic losses. Unfortunately for those simple models as soon as the quasi-static external conditions expire, huge differences appear. Small improvements can be obtained by adding to this lump model a simple dynamic contribution, product of a damping constant to the time domain derivation of the induction field B. This product is analogous to an equivalent excitation field H. Here again, even if this adjunction provides a relative improvement, correct simulation results are obtained on a narrow frequency bandwidth. It seems that a simple viscous losses term $\frac{dP}{dt}$ leads to an overestimation in the high frequency part of the magnetization hysteresis loop area versus frequency curve. Another correction of the lump model must be done to reach correct simulation results on a large frequency bandwidth. A mathematical operator dealing with the low frequency and the high frequency component in a different way than a straight time derivative is required. Such operators can be found in the framework of fractional calculus; they are the so-called non entire derivatives or fractional derivatives. Fractional derivation generalizes the concept of derivative to complex and non-integer orders. Fractional time derivative $\frac{dB^nB}{dt^n}$can be added in our lump model as referred by Grünwald Letnikov or Riemman-Liouville definitions [8]. Both of them are particular cases of a general fractional order operator namely, the first one represents the n order derivative, and the other one represents the n fold integral. In this sense, the class of functions described by the Riemman Liouville definition is broader (function must be integrable) than the one defined by Grunwald and Letnikov. However, for a function from the Grunwald-Letnikov class, both definitions are equivalent. In the present paper, we use the Riemman Liouville form for n ∈ [0,1].

$$\frac{d^n f(t)}{dt^n} = D_t^n f(t) = \frac{1}{\Gamma(1-n)} \frac{d}{dt} \int_{-\infty}^{t} (t-\tau)^{-n} f(\tau) d\tau \qquad (1)$$

where Γ is the Euler gamma function. According to Eq. 1, the fractional derivative of a function $f(t)$can also be considered as the convolution of a $f(t)$ function and$\frac{t^n}{\Gamma(1-n)}$. In that case n is the order of the fractional derivation. The additional time derivative present in the formula coincides with the occurrence of positive argument of the gamma function, Γ(.), leading to its convergence to a finite value. It is obvious that fractional derivative includes memory of the previous states. Fractional derivative is introduced in the lumped quasi-static hysteresis model through a dynamic contribution. The term $\rho \cdot \frac{dB}{dt}$ is replaced by $\rho \cdot \frac{d^nB}{dt^n}$, and this contribution is then added to the quasi-static contribution, Eq. 2.

$$\rho . \frac{d^n B(t)}{dt^n} = H_{dyn}(t) - f_{static}^{-1}(B(t)) \qquad (2)$$

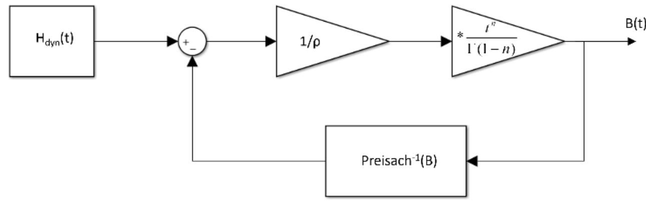

Fig. 1. Fractional dynamic lump hysteresis model.

III. Simulation Results, Comparison Measure/Simulation, and Conclusion

Specific experimental set up has been carried out in order to validate these simulation models. The sample tested is a low cobalt iron electrical alloy referenced SV142b. Both thickness and width are 5 mm; its conductivity is 1.4×10^7 (Ωm)$^{-1}$. Due to the high electrical conductivity of the test sample, even for relatively weak variation of excitation frequencies, large variation of cumulative magnetic losses appear.

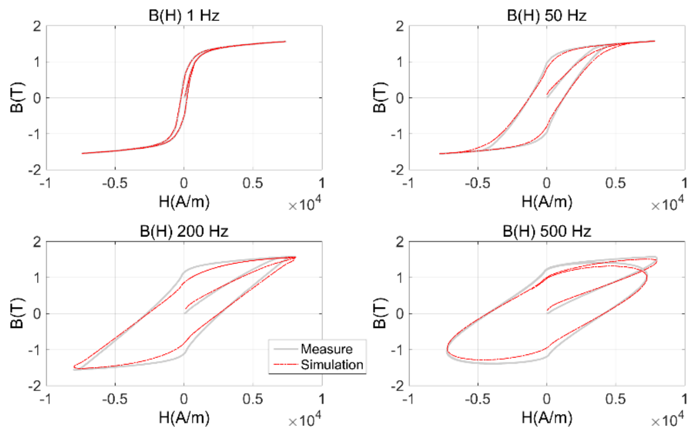

Fig. 2. Comparison simulation/measure for increasing frequency condition.

As illustrated in Figure 2, the fractional lump model gives precise macroscopic behavior *B(H)*, it leads to a very accurate formulation of the problem with a high reduction of the complexity and of the simulation times.

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